Let's get started with...

Logic!

Logic

- Crucial for mathematical reasoning
- Important for program design
- Used for designing electronic circuitry

- (Propositional)Logic is a system based on propositions.
- A proposition is a (declarative) statement that is either true or false (not both).
- We say that the truth value of a proposition is either true (T) or false (F).
- Corresponds to 1 and 0 in digital circuits

The Statement/Proposition Game

"Elephants are bigger than mice."

Is this a statement?	yes
Is this a proposition?	yes
What is the truth value of the proposition?	true

The Statement/Proposition Game

"520 < 111"

Is this a statement? yes

Is this a proposition? yes

What is the truth value of the proposition? false

The Statement/Proposition Game

"y > 5"

Is this a statement? yes

Is this a proposition? no

Its truth value depends on the value of y, but this value is not specified.

We call this type of statement a propositional function or open sentence.

The Statement/Proposition Game

"Today is January 27 and 99 < 5."

Is this a statement? yes

Is this a proposition? yes

What is the truth value of the proposition? false

The Statement/Proposition Game

"Please do not fall asleep."

Is this a statement? no

It's a request.

Is this a proposition? no

Only statements can be propositions.

The Statement/Proposition Game

"If the moon is made of cheese, then I will be rich."

Is this a statement? yes

Is this a proposition? yes

What is the truth value of the proposition? probably true

The Statement/Proposition Game

"x < y if and only if y > x."

Is this a statement? yes

Is this a proposition? yes

… because its truth value does not depend on specific values of x and y.

What is the truth value of the proposition? true

Combining Propositions

As we have seen in the previous examples, one or more propositions can be combined to form a single compound proposition.

We formalize this by denoting propositions with letters such as p, q, r, s, and introducing several logical operators or logical connectives.

Logical Operators (Connectives)

We will examine the following logical operators:

- Negation (NOT, $\neg$)
- Conjunction (AND, $\wedge$)
- Disjunction (OR, $\vee$)
- Exclusive-or (XOR, $\oplus$)
- Implication (if – then, $\rightarrow$)
- Biconditional (if and only if, $\leftrightarrow$)

Truth tables can be used to show how these operators can combine propositions to compound propositions.

Negation (NOT)

Unary Operator, Symbol: $\neg$

P	$\neg$ P
true (T)	false (F)
false (F)	true (T)

Conjunction (AND)

Binary Operator, Symbol: $\wedge$

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (OR)

Binary Operator, Symbol: $\vee$

P	Q	P $\vee$ Q
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive Or (XOR)

Binary Operator, Symbol: $\oplus$

P	Q	P⊕Q
T	T	F
T	F	T
F	T	T
F	F	F

Implication (if - then)

Binary Operator, Symbol: $\rightarrow$

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T